

POTW solution 7/11 - 7/18

The key idea is to recognize the similarity of the equations of heat flow with that of electrostatics.

We have $\vec{h} = -k \vec{\nabla} T$

and $\vec{\nabla} \cdot \vec{h} = S$

where k is the thermal conductivity and S is the heat generated per unit volume per second.

letting $\vec{h} \rightarrow \vec{E}$, $T \rightarrow \phi$

and $S \rightarrow \frac{\rho}{\epsilon_0}$

$k \rightarrow$ The dielectric const. ϵ_0

we obtain the equations of electrostatics.

Notice that in the electrical analogy we have a point charge in a half space medium of dielectric constant k , and the other half space has a dielectric constant of 0.

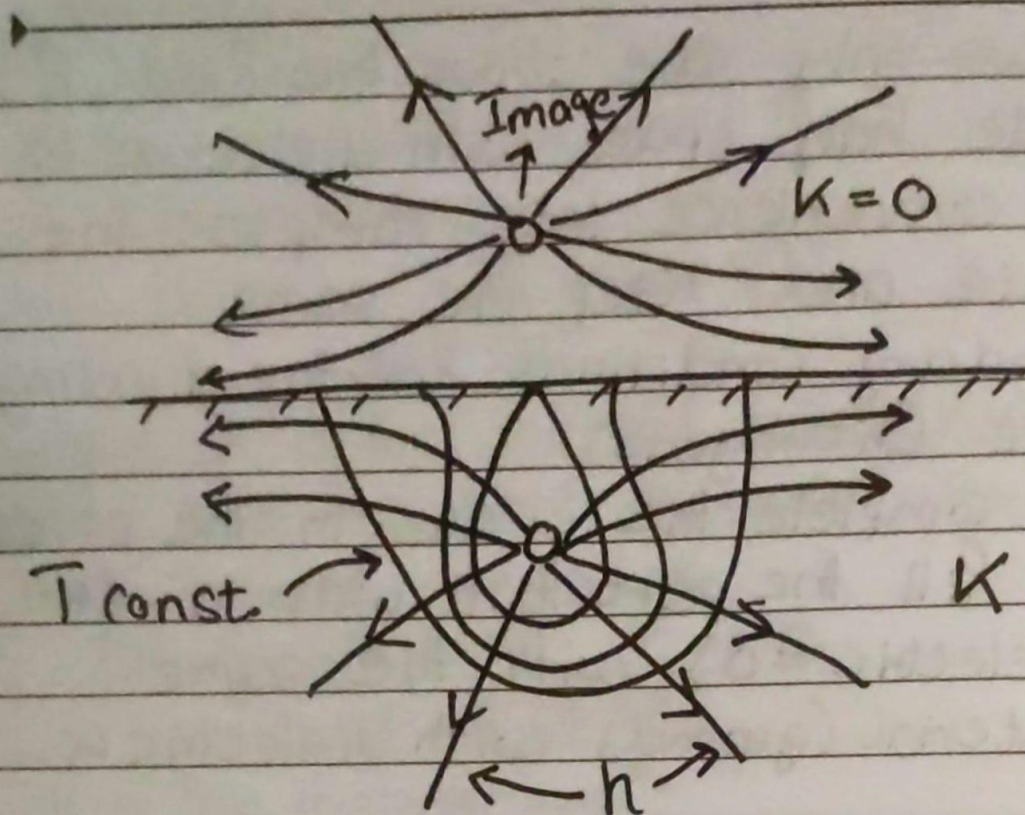
Thus, \vec{h} must be tangent to the plane dividing the ground and the air.

So we only care about the field (\vec{E}) inside half space with dielectric K , for convenience let's complete the space and keep the same boundary conditions for the dividing plane. (\vec{E} tangent)

We complete the space in the sense we fill the part that was air ($\epsilon = 1$, dielectric = 0) with the same material (ground) with dielectric K .

Notice if we pick an image charge for our point charge, reflected about the dividing plane, of the same strength and sign as our original, the field will satisfy our boundary conditions.

Temp.
From electrical analogy the field of a point heat source is given by
$$T = \frac{G}{4\pi K r}$$
 where G is the energy per second by the source.



Thus the temperature at any point of our concerned space is easily calculated by:

$$T = \frac{G_1}{4\pi k w_1} + \frac{G_1}{4\pi k w_2}$$

where w_1, w_2 respectively are the distances from the original and image source.

Note: For a discussion of the same in Feynman's words see FLP vol II lec 12-2