SPS Problem of the Week 11/14/2020-11/21/2020

Problem 1. Let $\mathbb{Z}^n$ be the integer lattice in $\mathbb{R}^n$. Two points in $\mathbb{Z}^n$ are called neighbors if they differ by exactly 1 in one coordinate and are equal in all other coordinates.
For which integers $n \geq 1$ does there exist a set of points $S \subset \mathbb{Z}^n$ satisfying the following two conditions?

1. If p is in S, then none of the neighbors of p is in S.
2. If $p \in \mathbb{Z}^n$ is not in S, then exactly one of the neighbors of p is in S

Note: This is a hard problem, draw it out, try to figure it out for 2 and 3 dimensions and for the general case, a hint is to use a construction which uses modular arithmetic, the arithmetic of remainders.