SPS Problem of the Week 01/24 - 01/31

Problem:

Let n be a positive integer. Harry has n coins lined up on his desk, each showing heads or tails. He repeatedly does the following operation: if there are k coins showing heads he flips the kth coin over; otherwise he stops the process. (For example, the process starting with THT would be $THT \to HHT \to HTT \to TTT$, which takes three steps.)

Letting C denote the initial configuration (a sequence of n H's and T's), write $l(C)$ for the number of steps needed before all coins show T. Show that this number $l(C)$ is finite, and determine its average value over all possible 2^n initial configurations C.